OPTIMAL FISCAL STRATEGY FOR OIL EXPORTING COUNTRIES

By
Eduardo Engel and Rodrigo Valdés

Optimal Fiscal Strategy for Oil Exporting Countries

Eduardo Engel and Rodrigo Valdés[1]

January 27, 2000

Abstract

This paper develops simple guidelines for fiscal policy in oil producing countries, focusing on three issues: intergenerational oil distribution, precautionary saving, and adjustment costs. The paper presents a framework to analyze how the revenue generated by an exhaustible source of wealth that belongs to the government should be distributed between current and future generations. This framework is used to show the strengths and limitations of existing answers, which motivates a new approach for dealing with this question. The paper derives simple, closed form approximations to the optimal level of government expenditure when an important part of government revenue is generated by an uncertain and exhaustible natural resource such as oil. Price uncertainty, budget uncertainty, and the (possibly asymmetric) costs of adjusting expenditure levels are considered.

Key words: optimal fiscal policy, stabilization fund, intergenerational oil distribution, precautionary saving, adjustment costs, exhaustible natural resources, optimal government expenditure, price uncertainty, budget uncertainty, oil exporting countries

JEL classification: E21, E61, E62, H50, H60, O16, O23, O53, Q33, Q38

[1]Engel: Center for Applied Economics (CEA), Department of Industrial Engineering, University of Chile and NBER. Valdés: Central Bank of Chile. E-mail: eengel@dii.uchile.cl and rvaldes@minhda.cl. This paper was prepared for the Middle Eastern Department at the International Monetary Fund. The authors thank David Burton, Paul Chabrier, Ugo Fasano, Antonio Rangel and seminar participants at the IMF for helpful comments and suggestions. All remaining errors are our responsibility. This paper presents the views of the authors and does not represent in any way positions or views of the Central Bank of Chile.

and none significantly better than a geometric random walk where the forecast of future prices is equal to the current price.

This paper studies fiscal strategy from a normative point of view. Our purpose is to develop a set of rules that can improve welfare assuming a particular set-up. The paper does not study problems of fiscal policy sustainability,[3] since we assume throughout that the government intertemporal budget constraint is always satisfied, thereby ruling out Ponzi schemes.

The paper is organized as follows. Section 2 presents a framework to discuss the intergenerational oil distribution problem. Section 3 discusses the intuitions behind the design of optimal fiscal policy. Section 4 evaluates two existing approaches to the problem of intergenerational distribution and proposes a new one. Section 5 characterizes the stochastic process of oil prices. Section 6 derives policy guidelines based on precautionary saving and adjustment costs. Section 7 discusses the role of stabilization funds. Finally, section 8 concludes.

2 Framework

In this section we provide an organizing framework to analyze the following question:

> *How should the revenue generated by an uncertain source of wealth that belongs to the government, such as oil in the case of oil exporting countries, be spent and distributed between current and future generations?*

An answer to this question has important policy implications, since it brings with it a prescription for optimal fiscal policy, providing guidelines for managing variables such as government deficits, government expenditures, taxes, the current account and stabilization funds.

The standard economic framework for analyzing the normative question we are concerned with is the following one:

(a) Choose a Social Welfare Function (SWF).

(b) Decide the set of policy instruments available to the government and the constraints it faces.

(c) Choose a set of assumptions (and constraints) for private sector behavior.

(d) Find the values of the policy instruments considered in (b) that maximize the SWF specified in (a) subject to the behavioral assumptions made in (c). We refer to this problem as the *Optimal Consumption Problem.*

[3]See., e.g., Liuksila et al. (1994) for a discussion about fiscal sustainability in oil producing countries.

For simplicity we assume that c_G is determined by the government's current expenditure level. A more realistic assumption, which we may explore in future versions of this paper, is that it also depends on past government expenditures.[5]

2.1.2 Social Welfare Functions

A typical SWF (at time 0) is (the expected value of) a function of the instantaneous utilities of present and future generations:

$$\mathcal{W} = \mathrm{E}_0[W(u_0, u_1, u_2, ...)]. \tag{3}$$

Where E_0 denotes the expected value, conditional on the distributions of unknown quantities (such as future oil prices) based on information available at time $t = 0$ and u_0, u_1, u_2, ... denote the instantaneous utilities at times 0, 1, 2, ... The function W is increasing in all its arguments. It also exhibits decreasing marginal returns in all its arguments.

The quantities $u_0, u_1, u_2, ...$ in (3) may also be interpreted as the utilities of a representative consumer in consecutive years (instead of generations).

The most commonly used SWF are the following:

Utilitarian SWF

A SWF W is *utilitarian* (or of the Bentham-Ramsey type) if it is a weighted sum of the utility of present and future generations:

$$W(u_0, u_1, u_2, ...) = \sum_{t \geq 0} \beta^t N_t H(u_t). \tag{4}$$

The parameter β denotes the subjective discount rate. This value is close to but smaller than one; the smaller it is, the larger the degree of impatience in the SWF.

N_t denotes the population at time t. The social welfare function grows in proportion to the population. We will assume that $N_t = (1+n)^t$, so that the population grows at a constant rate n.

The function H is a standard utility function, increasing, with decreasing marginal utility. A particularly useful case of (4) is:

$$H(u) = u^{1-\rho}/(1-\rho), \tag{5}$$

with $\rho > 0$.[6] This is the Constant Elasticity of Substitution (CES) utility function: $1/\rho$ denotes the elasticity of substitution of consumption at different moments in time. Furthermore, if there is

[5] This requires distinguishing between government expenditures on the public good and government investments that produce a future flow of the public good.

[6] If $\rho = 1$ we define $H(u) = \log(u)$.

2.2 Policy Instruments

A variety of policy instruments may be available to governments when implementing fiscal policies. Savings and debt, taxation, investment, and stabilization funds are among those most relevant for the problem considered in this paper.

2.2.1 Privatization

The government of an oil exporting country could consider the possibility of privatizing the state-owned oil monopoly, as was done, for example, recently in Argentina.[7] In this paper we rule out this possibility. One reason for doing so is that the government may be unable to commit credibly not to expropriate the privatized firm. Yet even if oil is fully privatized, the fiscal authority still faces the problem of how to distribute the proceeds across generations. What privatization does is reduce uncertainty with respect to initial wealth, besides likely efficiency gains which go beyond the scope of this paper.

Even though we do not consider privatization in the set of feasible policy instruments, we extensively use the *possibility* of future privatization as a convenient short-cut to derive approximations to the solution of the optimal consumption problem under uncertainty.

2.2.2 Savings

Governments can hold financial assets to finance future expenditures. We denote the gross real interest rate accrued per period for these savings by R, and assume that it is known and constant over time.

2.2.3 Debt

Governments incur debt to finance current consumption, investment and interest payments on previously incurred debt. The interest paid varies over time, both due to international and local factors. Nonetheless, interesting insights can be obtained even if the simplifying assumption of a fixed real interest is made. This assumption is justified by noting that oil prices are considerably more volatile than interest rates. Furthermore, we ignore any difference between the interest rate paid on debt and that accrued to savings, and denote both gross rates by R.

The following equation describes the evolution of government financial assets, when savings and debt at a gross interest rate of R are possible:

$$F_{G,t+1} = R(F_{G,t} + Y_{G,t} - C_{G,t}). \tag{11}$$

[7]It should be noted, though, that oil is not one of Argentina's main exports.

Government expenditures face an intertemporal budget constraint, that is they must eventually be financed through taxes or other sources of government income. This budget constraint, as of period 0, states that the present value of government incomes must equal the present value of government expenditures, that is:

$$F_{G,0} + \sum_{t \geq 0} R^{-t}[Y_{G,t} + \Gamma_t] = \sum_{t \geq 0} R^{-t} C_{G,t}. \tag{13}$$

2.2.6 Stabilization Funds

A stabilization fund saves and spends money with the objective of stabilizing a specific aggregate variable, such as overall government expenditures or government expenditures financed from the profits generated by a government owned primary commodity such as oil. The fund is held in liquid assets and incentives must be put in place to prevent the assets from being spent due to political pressures.

A well designed stabilization fund should be closely related to the solution of a problem of the sort posed at the beginning of this section. The savings/spending rule should be such that, in combination with other sources of government savings/credit, it implements the optimal fiscal strategy. Furthermore, a government may value liquidity *per se*, in which case having a stabilization fund may be desirable even if the government's net financial position is negative.

2.3 Private Sector

An important issue regarding private sector behavior is whether there is a bequest motive or not. The assumption of no bequest motive (or, more generally, of a weak bequest motive) is implicit in the intergenerational equity question central to this paper, for otherwise no government intervention would be needed to ensure that future generations benefit government owned wealth. If current generations do not care for their descendants, the private sector will not save for future generations and, given the opportunity to do so, will spend all the government owned wealth.[9]

The private sector also participates in the production of goods and services in markets which are assumed competitive. These goods and services may be consumed locally or exported. The private sector also has access to international finance for investment projects within the country.

The private sector also maximizes a welfare function, which even though qualitatively similar to the SWFs considered earlier in this section, may differ in some fundamental ways. An important difference we will encounter in most cases is that the time horizon considered by private agents is considerably shorter than that considered by the government's SWF. This is due to our assumption that private agents do not want to leave inheritance to their descendants.

[9]Strictly speaking this assumes no uncertainty about an individuals life span. If individuals do not know when they will die, they may die with positive net assets but this effect is typically small and will be neglected.

properties, such as a degree of risk aversion that *increases* with consumption levels and the implication that the optimal consumption path does not depend on the variance of income.

Another intuition that follows from consumption smoothing with uncertain income is that the government should react differently to transitory and permanent changes in income. A transitory positive shock to income should increase consumption only by the annuity value of the income shock. By contrast, a permanent increase should be met by a one-for-one reduction of consumption. For example, the increase in the price of oil following the invasion of Kuwait by Iraq in August of 1990 was clearly transitory. By the time the oil price had returned to its pre-invasion levels (in mid 1991), the rule described above can be used to spend the windfall generated by the price increase.

More generally, if income follows an autoregressive process with first order correlation ψ, which therefore also captures the degree of persistence of income shocks, the fraction of the current shock to income that should be spent is $(R-1)/(R-\psi)$.[10] The case $\psi = 0$ corresponds to i.i.d. (and therefore transitory) shocks while $\psi = 1$ corresponds to the case where income follows a random walk (permanent shocks).

In practice it is often not easy to determine the extent to which a change in income is permanent or transitory. Most shocks can be thought of as having both a permanent and a transitory component. In Section 6 we review recent econometric developments that can be used to accomplish this decomposition, concluding that a geometric random walk appears as a sensible description for the oil price.

Furthermore, because oil is an exhaustible resource, even permanent price shocks have only a transitory effect on income. The transitory component of the shock is more important the shorter the expected duration of the resource.

3.2 Precautionary Saving

A fundamental intuition underlying savings behavior is that an increase in risk should increase current savings and decrease current consumption. This is known as the *precautionary saving* motive, see Leland (1968). The *consumption smoothing* intuition does not incorporate this idea, since it prescribes that the current annuity value of expected wealth should be spent every year, regardless of the degree of uncertainty associated with this wealth.

To capture the precautionary savings motive, we must consider more realistic instantaneous utility functions than the quadratic case. This typically comes at the price of not having an explicit expressions for optimal consumption,[11] and numerical methods must be used to determine the optimal plan (as in Zeldes [1989], Deaton [1991] and Carroll [1992]).

[10] See, for example, Flavin (1981).

[11] Caballero (1990) finds a particular case where an explicit expression for optimal consumption can be derived. Yet he assumes constant *absolute* risk aversion, which also has unappealing properties.

not hold and all sources of finance should be used in such a way that the marginal distortion they introduce is the same over time and across financing instruments. This result is referred to as "tax-smoothing", see, for example, Barro (1979).

4 Intergenerational Redistribution

In this section we discuss the problem of how to distribute oil wealth across generations. We analyze the degree to which two well known approaches to optimal fiscal policy correspond to particular cases of the framework developed in Section 2 and offer a new approach to deal with this problem.

4.1 Benchmark Model

The following model will be a useful benchmark throughout this section.

(a) **Social welfare function**: Utilitarian with constant elasticity of substitution across time $(1/\rho)$. The initial population is normalized to one and grows at a constant rate n. The time horizon is infinite and there is no income uncertainty. Then (4) becomes:

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t (1+n)^t u_t^{1-\rho}. \tag{14}$$

The instantaneous utility has consumption of the public and private goods as separate arguments and the elasticity of substitution between both consumption goods is constant $(1/\gamma)$ as in (2).

(b) **Policy Instruments**: The government is the only provider of the public good, which it finances with taxes, debt and proceeds from the sales of the government owned natural resource (oil in what follows). Oil income in period t is denoted by $Y_{G,t}$; it is known with certainty and determined exogenously.

The government collects taxes and makes transfers to the private sector without generating any distortions in doing so. The government may also save and borrow at the international gross rate R. The only constraint it faces in setting taxes and borrowing is its intertemporal budget constraint (13). Initially it holds financial assets equal to $F_{G,0}$.

(c) **Private Sector**:

Consumers live for one period and have no bequest motive; it follows that the private sector holds no assets or debt. Private sector production in period t is exogenous and equal to $Y_{P,t}$.

The absence of bequests and the assumption that individuals live for one period imply that the private sector will accumulate no assets. Hence $F_{G,t} = F_t$ and the current account surplus is equal to the government's total (including interest receipts) surplus. Furthermore, optimal per capita taxes, τ_t, are equal to:

$$\tau_t = c_{P,t} - y_{P,t}. \tag{21}$$

4.1.1 Examples

Example 4.1 (Constant Non-oil Production) *We assume no population growth ($n = 0$), $R = 1.06$, $\beta R = 1$,*[13] *and no initial financial assets ($F_0 = 0$). The optimal mix of the public and private goods requires that the former represent 20% of total consumption.*[14]

Initial oil production, which accrues to the government, accounts for 80% of GDP, while the remaining 20% is produced by the private sector. Oil production remains constant (in real terms) for 25 periods, moment at which oil reserves are exhausted. Production in the non-oil sector remains constant indefinitely.

Figure 4.1 shows the evolution of consumption, financial assets (as a fraction of non-oil GDP), and the current account (also as a fraction of non-oil GDP). The first two series are divided by 100 and 50, respectively. It can be seen that consumption remains constant and equal to the annuity value of initial wealth (both from the oil and non-oil sectors). During the "boom years" of oil production, assets are accumulated (by the government) to maintain a level of consumption above production once oil is exhausted. During the boom years we also observe a positive and, due to interest payments, increasing current account surplus, which turns into a constant deficit once oil is exhausted. Since oil revenues can finance more than the optimal level of the public good, the government transfers a fixed amount (not shown in the figure) to every generation.

It is interesting to note that if $\beta R < 1$ (impatient individuals), the consumption path will be downwards sloping instead of constant, since individuals want to consume more and save less today. If this effect is large enough, there may be no initial current account surplus, as individuals spend more than the sum of their private income and the current oil income. ∎

Example 4.2 (Increasing Non-oil Production) *Assume now that, instead of remaining constant, non-oil production grows 2% per period forever. The remaining assumptions are the same as in the previous example.*

Figure 4.2 shows the evolution of the same three variables considered in Figure 4.1, with the same normalizing constants. It also shows the path of optimal taxes (as a fraction of non-oil GDP). Consumption is constant, at a level 12.3% higher than in Figure 4.1, reflecting the fact that non-oil

[13] This assumption makes the value of ρ irrelevant in this problem.

[14] This is equivalent to having $k^{1/\gamma} = 4$.

revenue will be saved and consumed. To make this decision based on intergenerational equity considerations, it is necessary to determine the permanent rent available from hydrocarbon exploitation. This rent represents the level of public consumption that can be currently enjoyed without increasing the country's debt and depleting its wealth."[15]

This approach can be rationalized within the framework of Section 2 as follows:

(a) **Social welfare function**: The difference with the BM is that the instantaneous utility function only depends on consumption of the public good.[16]

(b) **Policy Instruments**: The difference with the BM is that the government cannot collect taxes.

(c) **Private Sector**: The private sector does not appear, at least explicitly, in the problem.

The Permanent Oil Income Model (POIM) considers the problem of spending the government owned oil as if it were totally unrelated to the private sector's consumption of private goods. The solution to the problem is obtained by substituting total initial *government* wealth for total wealth in (17):

$$\mathcal{W}_{G,0} \equiv F_{G,0} + \sum_{s \geq 0} R^{-s} Y_{G,s}. \tag{22}$$

We then have:

$$\begin{aligned} C_{G,0} &= (1-\tilde{\alpha}) R \mathcal{W}_{G,0}, & (23)\\ c_{G,t+1} &= [\beta R]^{1/\rho} c_{G,t}. & (24) \end{aligned}$$

If $\beta R = 1$, the right hand side of (23) (divided by period 1 population) is *permanent oil income*, that is, the highest per capita consumption level from oil resources that can be maintained indefinitely, thereby justifying the name of the model.

The POIM can be used to rationalize the often mentioned criterion of intergenerational fairness according to which oil wealth (either in absolute or per capita terms) should be kept constant. Equations (23) and (24) imply that per capita government wealth, which in this model corresponds to oil wealth, remains constant along the optimal consumption path only if $\beta R = 1$.[17] If $\beta R < 1$, it is optimal for society to deplete oil wealth as time goes by. It also follows from (23) and (24) that *total* oil wealth remains constant along the optimal consumption path only if $\beta R(1+n)^{\rho} = 1$. If $n > 0$ this requires a relatively impatient society, since $\beta R < 1$.

[15] Quoted from Fasano (1999, p. 1).

[16] That is, it corresponds to the particular case of (2) where $k = 0$.

[17] To derive this result evaluate (23) at t and $t+1$, instead of $t = 0$, and equate the corresponding ratio to that obtained from (24).

intergenerational wealth transfers. It follows that the solution to the POIM is the same as that of the BM. Disregarding consumption of the private good when choosing the optimal consumption path is of no consequence in this case.

The equivalence between both optimal paths breaks down if we assume $\beta R < 1$. In this case, the increasing consumption path prescribed by the BM will be steeper than the one prescribed by the POIM. ∎

Example 4.4 (Increasing Non-oil Production) *We modify the previous example by assuming that non-oil GDP grows at 2% per period. Optimal consumption of the public good is constant and total consumption increases over time at the same speed as private income. The optimal consumption path is the path of private income shifted by the permanent oil income. The optimal consumption path differs significantly from that obtained in Example 4.2. The government accumulates financial assets while oil is extracted, but asset accumulation is considerably less than in the solution to the BM, since the government is not allowed to use taxes to make intergenerational wealth transfers.* ∎

It follows from both examples above that if oil wealth is front loaded and individuals are not very impatient, the country should save part of the resource proceeds. The counterpart of these savings is a persistent fiscal and current account surplus for some time. This is the main conclusion in Alier and Kaufman (1999), who work with a model that has the SWF of the Benchmark Model but assume constant and exogenous taxes, thereby avoiding intergenerational wealth redistribution. The latter assumption makes their problem equivalent to our POIM, with identical policy prescriptions and limitations.[19]

4.3 A New Approach

Both models discussed in the previous subsections have serious shortcomings. The Benchmark Model allows for intergenerational wealth transfers which we do not observe even in the absence of oil wealth. On the other hand, the POIM avoids intergenerational transfers by ruling out government policies that benefit all generations (as viewed from the BM). The Benchmark Model's SWF is more appealing than that of the POIM, since individuals benefit both from consumption of the private and public goods. Regarding instruments, the BM has more than we would like, while the POIM eliminates unattractive instruments (intergenerational wealth transfers) at the cost of ruling out appealing policy alternatives.

The challenge therefore is to limit the policy instruments available to the government in the BM in such a way that the attractive properties of both models can be recovered. We propose

[19]Their generations live for two periods, yet no additional insight is gained from this assumption. Also, the mix of public and private good provided is typically not optimal.

Those that benefit the most are those that would have been poorest without oil wealth—generations that expected relatively high private incomes do not benefit at all. ∎

Example 4.7 (Decreasing Non-Oil Income) *Figure 4.4 shows what happens when non-oil income* decreases *by 2% during the first 50 periods, and remains constant thereafter.*[21] *The behavior of the optimal consumption path in the BM and POIM are qualitatively similar to those described in the previous example. In the case of the CNM, optimal consumption decreases initially, being equal to non-oil income during this phase. Eventually (period 13 in the figure) it stops decreasing and remains constant thereafter. By contrast with Example 4.6, in this case the optimal consumption path of the CNM is fiscally more conservative than that of the POIM. It prescribes not spending oil related wealth during early years, saving it to help those who expect to be worse off in the future. Only in period 13 the CNM recommends to begin spending oil wealth to help maintain the highest consumption level compatible with the restriction of not leaving any generation worse off than it would have been without oil. It is also interesting to note that in this example the consumption path of the Benchmark Model is the one that is most conservative from a fiscal point of view. It taxes heavily the initial generations to finance a constant level of consumption for everybody.* ∎

The following general result for the optimal consumption path under CNM is presented in the Appendix (Propositon A.2). It assumes no income uncertainty and $\beta R = 1$. Under these assumptions, the optimal consumption path for the CNM can be found as follows: First, the generations are ordered according to their utility in the non-oil scenario. Next, oil wealth is used to raise the income of the poorest generation until it equals that of the second poorest. If this does not exhaust the oil wealth, the income of the two poorest generations is raised until it equals that of the third poorest. And so on until no oil wealth remains to be distributed. If oil wealth is large enough so that the income of all generations can be brought to the level of the richest generation (in the scenario without oil), the constraint that differentiates the CNM from the BM is not be binding and both optimal consumption paths are the same (constant, equal to the annuity value of total wealth). Otherwise, the richest generations do not benefit from the oil wealth.

5 Oil Related Uncertainty

Characterizing the stochastic process that oil prices follow and evaluating the possibility of forecasting them are key ingredients when designing optimal fiscal policy rules for oil producing countries. For instance, recommendations regarding both the decision to adjust or finance a given price (terms of trade) shock and the design of an optimal oil stabilization fund depend of what is expected to happen with future prices, including their distribution. If each and every shock is regarded as

[21] The remaining parameter values are those of Figure 4.3.

For example, with several years of data, Videgaray (1998) finds mean reversion after allowing for a structural break in 1973.[25] Pindyck (1999) rejects the random walk null hypothesis using an ADF unit root test only after considering more than 70 years of data. Interestingly, he concludes that even with 120 years of data, permanent shocks do exist (although their size is considerably smaller than that of the transitory shocks). Finally, Bessembinder et al. (1995) find evidence of mean reversion using the future prices term structure.

The difficulty in rejecting the random walk hypothesis has led to more sophisticated models to describe oil prices. Rather than assuming reversion to a constant trend, Pindyck (1999) proposes a model in which both the constant and the trend are, in turn, non observable mean reverting stochastic processes. He estimates this model with a long sample of annual data using a Kalman Filter, and predicts prices 20 years ahead. Although no formal tests are provided, the forecasts appear to be better than those of a fixed trend AR(1) process. Of course, there is always the question of whether it is valid to use pre 1973 data to forecast future prices given the large structural break that took place at that time.

Schwartz (1997) also presents Kalman Filter estimates and formally compares the forecast capability of three alternative models for future and forward prices using high frequency data spanning 11 years. He considers a one factor model in which the (logarithm of the) oil price follows an AR(1) process, a two factor model in which the convenience yield is stochastic, and a three factor model in which a stochastic interest rate is also included. The estimation procedure he uses takes into account that the spot price, the convenience yield and the interest rate are not perfectly observable—thus the need of the Kalman Filter. The results he obtains indicate that including a second factor (the convenience yield) improves substantially the forecast capability of the model.

A simple random walk, an AR(1), and the models presented in Pindyck (1999) and Schwartz (1997) can be thought of as special cases of the following model:

$$p_t = \alpha_t + \delta_t \text{Trend}_t + \psi_t p_{t-1} + \varepsilon_t$$

where p_t is the log of the real oil price, α_t, δ_t and ψ_t are possibly stochastic parameters, Trend_t is a time trend and ε_t is a stochastic stationary shock.

A random walk with drift assumes α_t constant, $\delta_t = 0$, and $\psi_t = 1$ (as well as ε white noise). An AR(1) assumes a constant α_t, a constant $\psi_t < 1$ and (possibly) a positive δ_t.

More interestingly, Pindyck (1999) considers that both α_t and δ_t follow unobservable AR(1) stochastic processes with uncorrelated innovations. These processes are meant to represent reduced forms for the effects of demand, cost of extraction and available reserves shocks. Prices then would revert to a changing trend (level). Also, Schwartz (1997) considers the possibility that in his two

[25]He uses the Perron (1989) test which basically augments the standard Augmented Dickey-Fuller test to take into account structural breaks in levels and/or slope of a series.

5.2.2 Variance Ratio Test

The second type of test we consider to evaluate whether oil prices follow a non-stationary process is the Variance-Ratio (VR) Test. This test makes use of the linearly increasing volatility of a non-stationary process and evaluates whether the standard deviation measured at different horizons increases as predicted under the null of random walk. Furthermore, it gives a measure of the relative importance of transitory and permanent shocks.

In particular, the VR test calculates a statistic $J(s)$, $s = 1, 2, ..., S$ that has the following properties.[26] As the sample size becomes large and s increases the ratio $J(s)/s$ should converge to zero if the true process is stationary. If it does not converge to zero the process is non-stationary. Moreover, the value to which $J(s)$ converges represents the standard error for long term forecasts. These properties hold as long as the sample size is large and s is considerably smaller than this sample size.

Figures 5.1 and 5.2 present the results of VR tests for the log of the oil price for two samples: 1957.I–1998.IV and 1974.I–1998.IV. In both cases the statistic $J(s)/s$ does not converge to zero, showing that the shocks to the true process probably have some permanent effects. The size of these effects appears clearly smaller than the standard deviation of the innovations of a simple random walk estimated for each sample. This fact shows that shocks also have some transitory effects on prices, suggesting that it should be possible to do better, in terms of forecasting, than with a random walk.

One important limitation of these results is that the sample sizes we consider are not very large compared to s. In order to evaluate how this issue may affect the results the figures also present the results of a Montecarlo experiment considering a sample of equal size to what we consider in the calculations. These Montecarlo experiments are based on 1000 replications of a process that has the same standard deviation and parameters as the true data.

The results of these experiments show that, indeed, the small sample affects the performance of the test (for the sample sizes we consider). The statistic $J(s)/s$ for a true random walk decreases instead of converging to a flat value. At the same time, a true AR(1) does not converge to zero for the values of s we consider (although it does not converge to a positive value either). These results, however, do not change our general interpretation of the process. Because the sample statistic decreases faster than for the random walk, we conclude that shocks do not have full permanent effects. And because it tends to converge to a positive value, we conclude that shocks do not have transitory effects only.

[26] See Hamilton (1994) for further details.

this subsection we evaluate the out of sample forecast capabilities of 12 alternative linear models, a non-linear model, market future prices, and market forecasts.

We consider two alternative samples, one starting in 1974 and the other starting in 1986, and calculate the root mean square error (RMSE) of forecasts at 1 and 2 year horizons proceeding as follows. We estimate repeatedly each model using quarterly data (and weekly data in one case) ending in the second quarter of the years 1994 to 1998 and forecast out of the estimating sample. Then we compute the RMSE using the forecast errors at 1 and 2 years horizons. For each model we have 5 one-year ahead and 4 two-year ahead forecast errors.

The linear models we consider (for the logarithm of the real price of oil) are the following:

1. A random walk without drift.

2. A random walk with drift.

3. An ARIMA(2,1,2). This model is the equivalent of a random walk augmented by a stationary process for the error term ε_t.

4. Same as above with a dummy variable that take the value 1 during the invasion of Kuwait in 1991.

5. An AR(1) without drift (assuming that the process is stationary).

6. The permanent value of a Beveridge and Nelson decomposition of the series.[28]

Models 7 through 11 consider an AR(1) model with stochastic first-order autocorrelation, ψ_t, which is estimated using the Kalman Filter. The models differ in the assumptions they make on the process followed by ψ_t and whether they include a linear trend or not for the price process.

7. The price process has no trend and ψ_t follows a random walk with innovations orthogonal to those of the price process.

8. As 7 but with a trend in the price process.

9. The price process has no trend and ψ_t follows an AR(1) process with innovations that are orthogonal to oil price innovations (this model resembles model 2 of Schwartz, 1997).

10. As 9 but with a trend in the price process.

[28] The Beveridge and Nelson decomposition identifies that permanent component of a series as the long run value at which the series would tend if there are no further shocks. It predicts future prices using a rolling ARIMA model ([2,1,2] in this case).

6.1 Income and Budget Uncertainty

Income uncertainty—the risk about future income realizations—can be incorporated easily into consumption models. If the instantaneous utility is quadratic, we have certainty-equivalence, and the results obtained in Section 4.2 need to be modified only slightly. For example, equations (18) and (19) become:

$$\begin{aligned} c_0 &= (1-\tilde{\alpha})R\mathrm{E}_o[\mathcal{W}_0], && (25) \\ \mathrm{E}_t[c_{t+1}] &= [\beta R]c_t, && (26) \end{aligned}$$

where E_t denotes expectations based on information available in period t. That is, all that changes is that uncertain quantities are replaced by their expected values. Of course, as mentioned in Section 3.2, this solution has the awkward property that current savings do not depend on the variance of future income.

In the more appealing case of a CES instantaneous utility, there does not exist a simple expression for c_0. The solution has to be found resorting to numerical methods. We propose instead an approximation to the optimal solution that is transparent and easily implementable. Of course, because it is an approximation it does not correspond exactly to the optimal solution.

Our procedure is based on a counterfactual experiment in which consumption decisions are made knowing that oil risk is diversified away in the near future, say that the oil industry will be privatized. This procedure allows us to simplify the consumption problem by collapsing all future periods in a single period and treating the overall problem as a two-period problem. Furthermore, assuming that the variance of oil price shocks is small, we can write a closed-form solution for consumption as a function of that variance and initial conditions.

More precisely, consider the period t optimal consumption decision knowing that the oil industry will be privatized in period $t+1$. Because in period $t+1$ all income uncertainty is resolved, from that moment onwards the consumption problem is trivial: under the assumption $\beta R = 1$ the solution is to consume the sum of the annuity values of the privatization proceeds and the financial assets available at that time. Assume, further, that oil risk is fully diversifiable in the world economy, so that the privatization proceeds equal the expected NPV of oil GDP conditional of the oil price observed in $t+1$. As of period t, the privatization proceeds is a random variable that depends on the oil price process. Moreover, it depends on the expected path of future oil prices.

Consider now the comparison of the optimal consumption decision of period t, knowing the oil price of that period, both under certainty equivalence (CE) and the optimal consumption level (given the actual volatility of the price process). The plan under CE corresponds to the POIM solution. The difference between the two consumption levels measures the precautionary savings motive.

has mean $\mu_0 = \mu_{P,0} Q_0$ and variance $\sigma_0^2 = \sigma_{P,0}^2 Q_0^2$. Finally, assume that initial population is N_0 and growths at rate n.

Denote by $c_0(\sigma_v^2, \sigma_0^2)$ the optimal period 0 per capita consumption level considering both types of uncertainty.[32] In the Appendix (Lemma B.1) we show that if σ_v^2 and σ_0^2 are small, this solution can be approximated by:

$$c_0(\sigma_0^2, \sigma_v^2) \simeq [1 - \Delta_{BU} - \Delta_{IU}]\, c_0(0,0), \tag{29}$$

with

$$\begin{aligned} \Delta_{BU} &= -\frac{c^1(0,0)}{c_0(0,0)} \sigma_0^2, \\ \Delta_{IU} &= -\frac{c^2(0,0)}{c_0(0,0)} \sigma_v^2. \end{aligned}$$

Where $c_0(0,0)$ is initial consumption if there were no uncertainty and the superscripts denote derivatives with respect to argument j ($j = 1, 2$).

In general, both correction factors comprise two components. One captures the precautionary motive and, as expected, is positive, so that resulting consumption is smaller than it would have been in the absence of this motive. The second component corresponds to an income effect due changes in initial wealth associated with variations in σ_0 and σ_v. For example, if the price of oil follows a geometric random walk and the mean of the innovations v_t does not vary with σ_v, the present discounted value of oil income grows with σ_v at a rate $\frac{1}{2}\sigma_v^2$. On the other hand, if the drift of the random walk $-\frac{1}{2}\sigma_v^2$ the negative drift cancels the effect of volatility on wealth and there is no income effect. Choosing between both alternatives is equivalent to deciding whether $\mathrm{E}_t[P_{t+1}] = P_t$ or $\mathrm{E}_t[\log P_{t+1}] = \log P_t$, both cannot hold due to Jensen's inequality. Since forecasts based on the former are more precise and income effects can be much larger than what common sense would suggest,[33] we ignore income effects in what follows.[34]

Define φ as the present discounted value of future income $\sum_{t=0}^{T-1} \beta^t Y_{t+1}$. In the appendix we show that the correction factors Δ_{BU} and Δ_{IU} are given by:

$$\begin{aligned} \Delta_{BU} &= \frac{1}{2}(1+\rho) \frac{\beta(r-n)^2}{(1+n)N_0^2 c_0(0,0)^2} \left. \frac{\partial \mathrm{Var}_0\left(Y_0 + \mathrm{E}_1[\varphi]\right)}{\partial \sigma_0^2} \right|_{\sigma_v = \sigma_0 = 0} \sigma_0^2, \\ \Delta_{IU} &= \frac{1}{2}(1+\rho) \frac{\beta^3(r-n)^2}{(1+n)N_0^2 c_0(0,0)^2} \left. \frac{\partial \mathrm{Var}_0\left(Y_0 + \mathrm{E}_1[\varphi]\right)}{\partial \sigma_v^2} \right|_{\sigma_v = \sigma_0 = 0} \sigma_v^2. \end{aligned}$$

Where ρ is the coefficient of relative risk aversion. Both correction factors are proportional to the coefficient of relative prudence, $1 + \rho$.[35]

[32] c_0 also depends on μ_0 and F_0, but since these parameters remain constant in what follow they are omitted.
[33] Consumption after applying the correction factors cam be much *larger* than under certainty equivalence!
[34] Expressions that include the income effect may be found in Proposition B.1 in the Appendix.
[35] See Kimball (1990).

the parameters of the baseline example. In this case ψ ranges from 0.9 to 1. When $\psi < 1$ we use the formulae described in the Appendix. In all these cases we disregard any income effects arising from volatility by directly applying the correction factors to the zero variance consumption.

The results show that precautionary saving increases sharply with the persistence of shocks. When ψ is around 0.9, correction factors are almost one-tenth of what they are in the case of a random walk. Furthermore, this difference is clearly non-linear. When ψ is around 0.95, correction factors are about one-fourth of what they are when $\psi = 1$. ∎

This key role for shock persistence in determining the importance of precautionary saving has been noted before (see, e.g., Skinner, 1988). It follows from the high sensitivity of wealth uncertainty to the degree of persistence in shocks, particularly in the neighborhood of a random walk.

Example 6.3 (Precautionary Saving and Financial Assets) *Figure 6.2 shows the correction factors Δ_{BU} and Δ_{IU} for levels of initial financial assets F_0 and the parameters of the baseline example. We have scaled F_0 by initial production, so it ranges from -4 to 4.*

As expected, financial assets accumulation makes less important precautionary saving. Because a larger portion of future consumption is secure when a country has more financial assets, precautionary saving decreases with F_0. In the example at hand, the correction factors drop by almost one third when financial assets increase from zero to four years of income. A similar pattern arises if one assumes that $\psi = 0.9$, although in this case correction factors are considerably smaller. ∎

Example 6.4 (Precautionary Saving and Resource Duration) *Figure 6.3 shows the correction factors Δ_{BU} and Δ_{IU} for different time horizons for resource exhaustion and the parameters of the baseline example. T varies from 5 to 105.*

The results show that the correction factors increase quickly with T to stabilize around $T = 40$. The opposite happens if $\psi = 0.9$ (case not reported). The intuition for the result is the following. Given an extraction rate, a longer duration represents a higher initial reserve level of the resource. This, in turn, represents higher total wealth, and less initial financial assets relative *to total wealth. Thus, a longer duration produces an effect that is similar to having less financial assets. When $\psi < 1$, a longer duration has two effects. One the one hand, it produces the same effect of reducing the share of financial assets in total wealth. On the other hand, because $\psi < 1$, income that is very far in the future is almost secure income, having the same effect of a higher F_0. Figure 6.4 shows the correction factors for different T assuming the "intermediate" case $\psi = 0.99$. In this case the correction factors increase with T between 10 and 20-25 and decrease thereafter.* ∎

In deriving precautionary saving correction factors we have so far assumed that there is only one source of income, namely oil production. A more realistic representation of oil exporting countries should incorporate natural gas extraction. In order to do so we assume that the price of gas is linear where c^* denotes the solution to the problem above when $k = 0$ (see Proposition A.1) and $l^* = \log c^*$.

The second term in (32) captures the costs of adjusting while the first term corresponds to the welfare costs associated with deviating from the optimal expenditure level in the absence of adjustment costs.

As k, the constant $\tilde{k}$ can take two values, one for expenditure reductions, $\tilde{k}^-$, and another for expenditure increases, $\tilde{k}^+$.

Proposition C.2 in the Appendix shows that there exist constants α^- and α^+, both between zero and one, such that a good approximation for the logarithm of optimal consumption at time 0 incorporating adjustment costs, l_0, consists of adjusting *partially* toward $l^* \equiv \log c^*$. Thus:

$$l_0 - l_{-1} = \alpha(l^* - l_{-1}),$$

where α can take two values, one if consumption increases (α^+) and another when it decreases (α^-). The constants α^+ and α^- are decreasing functions of $\tilde{k}^+$ and $\tilde{k}^-$. The fraction of adjustment prescribed is larger when adjustment costs matter less. The adjustment speed also increases with ρ, since larger values of ρ imply a smaller elasticity of substitution of consumption over time and therefore a stronger incentive to smooth expenditure.

6.2.2 Eliciting Adjustment Costs

A key parameter in determining the velocity of the adjustment process is the size of adjustment costs. In Proposition C.3 in the Appendix we show that if a policymaker is indifferent between

- the adjustment cost associated this period with an *increase* in per capita expenditure of $100 \times s_a$ percent

and

- the welfare improvement, in the absence of adjustment costs, associated with a $100 \times s_{na}$ percent increase in per capita expenditure

then her value of $\tilde{k}$ is given by

$$\tilde{k}^+ \simeq \frac{2s_{na}}{\rho s_a^2}.$$

A similar comparison, with a *decrease* in per capita expenditure in the first statement, leads to an analogous expression for $\tilde{k}^-$.[36]

It is recommended that the value of s_{na} in the exercise described above be chosen neither too large (because the approximations involved become less precise) nor too small (because it is harder to make the comparison that is required). Suggested values are in the range from 0.05 to 0.20.

[36]The question in the second statement continues being posed in terms of an *increase* in per-capita expenditure.

which often leads to a solution far from optimal.[37] Behind this type of rule is the notion that policymakers are able to distinguish transitory from permanent price shocks. Given the evidence revisited in section 5, this clearly is a very strong assumption.

There are a few studies that have designed optimal stabilization funds using numerical procedures and the POIM as the benchmark problem. For example, Arrau and Claessens (1991), Kletzer, Newbery and Wright (1990), and the collection of papers in Engel and Meller (1993) design optimal funds under alternative assumptions. However, extending the POIM to incorporate precautionary motives may have unappealing consequences, since this model ignores the path of private income, and therefore its correlation with oil income. In deriving the approximation for precautionary saving presented in this paper we have assumed that this correlation is zero (private income is constant). This clearly is a strong assumption that should be relaxed in future research.[38]

To illustrate this point we refer to an example discussed in section 4.2 in which oil and non-oil income were assumed to be perfectly negatively correlated. The precautionary motive suggests that the government should, on average, spend less in every period than it would in the certainty-equivalence case. Yet these additional savings serve no purpose in this case, since there is no uncertainty in *total* income. In general, when private sector income is ignored, as in the POIM, precautionary saving could differ significantly from what they would be if uncertainty in total income were considered.

The stabilization fund that follows the set of prescriptions derived in this paper is not different from an otherwise standard stabilization fund used in several countries. The only key difference is that the set of rules is relatively more complex, which allows for the implicit solution to be closer to the optimal one. In principle, the stabilization fund in this model corresponds to financial assets F_t, and the set of rules may include intergenerational distribution, budget and income uncertainty and adjustment costs. Thus, if fiscal policy follows the strategy we recommend here, it will implicitly act as a stabilization fund. Of course, this fund could be explicitly setup, easing transparency and accountability. The rules for operating the fund will be the counterpart of the proposed fiscal strategy.

One important issue regarding actual implementation of the optimal fiscal strategy is the treatment of fiscal investment. The model presented here does not include an explicit role for investment and assumes that all positive NPV projects are developed (probably by the private sector). However, at the same time, we have excluded any secondary source of credit for the government in order to obtain the expected results from the proposed fiscal strategy. In this setup the results of the model can be associated to the maximum non-oil sector deficit that should be financed by

[37]For a criticism of the Chilean Copper Stabilization Fund along these lines see Basch and Engel 1993).

[38]The CNM is an attempt to incorporate non-oil income into the analysis, but it does so without considering the effects of uncertainty.

particular result closer to the optimal one. Of course, because they are approximations, they do not represent the optimal solution itself. Our current research is intended to evaluate how accurate are these proposed approximations, both the expansion around the certainty equivalence solution and the assumption of one-period-ahead diversification.

The paper has derived fiscal prescriptions both under the assumption that the oil price follows a geometric random walk process and a AR(1) process (in logs). However, the evidence revisited and new econometric evidence provided show that the geometric random walk assumption appears to be a more sensible representation. Yet it should be stressed that the framework we developed makes use of this assumption only partially. In the proposed setup, budget uncertainty allows us to include next year expected future price (more precisely, its mean and variance) which could be different from the actual current price. The random walk assumption is binding only two periods into the future.

Another important assumption behind the approach followed here to study the effects of uncertainty is that the POIM is an adequate description of the problem faced by the government. This is equivalent to assume that non-oil income is uncorrelated with oil (and gas) income. Future research should incorporate the possibility of a non-zero correlation between both types of income.

For simplicity, the proposed fiscal strategy was developed assuming an annual frequency, since we made the implicit assumption that the government could not revise the budget during the budget year.[41] This assumption can be easily relaxed reinterpreting the data frequency conveniently. Furthermore, without changing frequency, the model could be used during the current fiscal year if new information becomes available and the political process allows for adjustments in the budget. Yet such an exercise would necessarily have to be of the once-and-for-all type, since recurrent revisions would modify the model (or, at least, the appropriate data frequency).

Finally, the proposed approach has implicit a stabilization fund which could be explicitly setup for transparency and accountability purposes. There are two key ingredients for this fund to work properly. First, it should follow a set of accumulation rules that implement the proposed fiscal strategy. And second, it imposes strong restrictions to other forms of government financing so that what the fund accumulates actually reflects changes in the net fiscal asset position.

[41] The are good political economy arguments to maintain this procedure. In particular, there could be important asymmetries in the way the political process reacts to positive and negative shocks.

[16] Leland, H.E., "Saving and Uncertainty: The Precautionary Demand for Saving," *Quarterly Journal of Economics*, **82** No. 2, 465–473, 1968.

[17] Liuksila, C., A. García and S. Bassett, "Fiscal Policy Sustainability in Oil-Producing Countries", IMF Working Paper 94/137, November 1994.

[18] Michael, P., A.R. Nobay and D. Peel, "Transactions Costs and Nonlinear Adjustment in Real Exchange Rates: An Empirical Investigation," *Journal of Political Economy*, **105** No. 4, 862–879, 1997.

[19] Perron, P., "The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis," *Econometrica*, **57** No. 6, 1361–1401, 1989.

[20] Pindyck, R., "The Long-Run Evolution of Energy Prices," *Energy Jounral*, **20** No. 2, 1–27, 1999.

[21] Powell, A., "The Costs of Commodity Price Uncertainty," mimeo, Nuffield College, Oxford, 1990.

[22] Rotemberg, J., "Monopolistic Price Adjustment and Aggregate Output," *Review of Economic Studies*, **49**, 517–531, 1982.

[23] Schwartz, E., "The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging," *Journal of Finance*, **52** No. 3, 923–973, 1997.

[24] Skinner, J., "Risky Income, Life Cycle Consumption, and Precautionary Savings," *Journal of Monetary Economics*, **22** No. 2, 237–255, 1988.

[25] Videgaray, L. "The Fiscal Response to Oil Shocks," Unpublished PhD Dissertation, Massachusetts Institute of Technology, 1998.

[26] Zeldes, S., "Consumption and Liquidity Constraints: An Empirical Investigation," *Journal of Political Economy*, **97** No. 2, 305–346, 1989.

Proof From Lemma 1, the optimal choices of $c_{P,t}$ and $c_{G,t}$ must satisfy

$$u(c_{P,t}, c_{G,t}) = \left(\frac{(1 + k^{1/\gamma})^{\gamma}}{1-\gamma}\right)^{1/(1-\gamma)} c_t.$$

Substituting this expression for u_t in (33) completes the proof. ∎

Proposition A.1 *Consider the intertemporal consumption allocation problem*

$$\max_{c_t} \frac{1}{1-\rho} \sum_{t \geq 0} \beta^t (1+n)^t c_t^{1-\rho}$$

subject to the intertemporal dynamic budget constraint

$$F_{t+1} = R[F_t + Y_t - C_t], \tag{34}$$

where $C_t = c_t N_t$, $N_t = (1+n)^t$, $Y_t = Y_{P,t} + Y_{G,t}$, *and* $N_0 = 1$. *Initial assets* (F_0) *and the complete future stream of income,* $Y_0, Y_1, Y_2, \ldots$ *are known at time 0.*

Then optimal per capita consumption at time 0 is

$$c_0 = (1 - \tilde{\alpha}) \mathcal{W}_0$$

and the optimal consumption path satisfies

$$\begin{aligned} c_{t+1} &= [\beta R]^{1/\rho} c_t, & (35) \\ C_{t+1} &= \alpha C_t, & (36) \end{aligned}$$

where $\alpha = (1+n)[\beta R]^{1/\rho}$; $\tilde{\alpha} = \alpha / R$, *and*

$$\mathcal{W}_0 \equiv F_0 + \sum_{s \geq 0} R^{-s} [Y_{G,s} + Y_{P,s}].$$

Furthermore, the period t current account of this economy is given by

$$CA_t = \left[2 - \frac{1}{R}\right] (Y_t - C_t) + \left[1 - \frac{1}{R}\right] F_t. \tag{37}$$

Proof We first derive the slope of the consumption path and then the initial consumption level. Using $N_t = (1+n)^t N_0$ and $C_t = c_t N_t$ it is possible to rewrite the objective function as

$$\max_{C_t} \frac{1}{(1-\rho) N_0^{1-\rho}} \sum_{t \geq 1} [\beta (1+n)^{\rho}]^t C_t^{1-\rho}.$$

Using the dynamic budget constraint $F_{t+1} = R[F_t + Y_t - C_t]$ the problem becomes

$$\max_{F_t} \frac{1}{(1-\rho) N_0^{1-\rho}} \sum_{t \geq 0} [\beta (1+n)^{\rho}]^t [F_t + Y_t - \frac{F_{t+1}}{R}]^{1-\rho}.$$

3. *Use oil wealth to raise the income of generation k until it equals that of generation $(k+1)$ or until it is exhausted, whichever happens first.*

4. *If 3 does not exhaust the oil wealth, increase k by 1 and return to 3. Otherwise, the resulting distribution of oil wealth solves the CNM.*

Proof The algorithm ends because we assumed that the solution to the BM violates the constraints of the CNM. The remainder of the proof is straightforward. ∎

B Precautionary saving

The following results consider the setup described in section 6.1.1.

Lemma B.1 *Denote by $c_0(\sigma_0^2, \sigma_v^2; F_0, \mu_0)$ the solution for optimal per capita consumption as a function of initial financial assets and parameters characterizing the distribution of future income. In what follows F_0 and μ_0 remain fixed and are therefore omitted. Assuming $c_0(\sigma_0^2, \sigma_v^2)$ has continuous second order partial derivatives, we have that*

$$c_0(\sigma_0^2, \sigma_v^2) = [1 - \Delta_{BU} - \Delta_{IU}]\, c_0(0,0) + \mathcal{O}(\sigma^4), \tag{38}$$

with

$$\Delta_{BU} = -\frac{c^1(0,0)}{c_0(0,0)}\sigma_0^2, \tag{39}$$

$$\Delta_{IU} = -\frac{c^2(0,0)}{c_0(0,0)}\sigma_v^2. \tag{40}$$

Where the superscripts denote derivatives with respect to argument j $(j = 1, 2)$, $\sigma = \max(\sigma_v, \sigma_0)$, and $\mathcal{O}(\sigma^4)$ denotes a term of order σ^4.

Proof By continuous second order partial derivatives we mean that c_0^{11}, c_0^{22} and c_0^{12} are well defined and continuous. The result then follows from taking a first order Taylor expansion of $c_0(\sigma_0^2, \sigma_v^2)$ around $(0,0)$. ∎

Corollary B.1 *Assume that an increase in uncertainty (that is, either an increase in σ_0 or σ_v) does not affect initial wealth,[44] so that $c_0^{CE}(\sigma_0^2, \sigma_v^2) = c_0^{CE}(0,0)$, where c_0^{CE} denotes optimal per capita consumption under certainty equivalence and the arguments are the same as in the preceding proposition. Then*

$$c_0(\sigma_0^2, \sigma_v^2) = [1 - \Delta_{BU} - \Delta_{IU}]\, c_0^{CE}(\sigma_0^2, \sigma_v^2) + \mathcal{O}(\sigma^4), \tag{41}$$

with Δ_{BU}, Δ_{IU} and $\mathcal{O}(\sigma^4)$ defined above.

[44] This holds, for example, when the price of oil follows a geometric random walk with drift such that $E_t[P_{t+1}] = P_t$.

Next we spell out the details. Since all income uncertainty is eliminated in period 1, optimal consumption at that point in time will be equal to certainty equivalent consumption, so that (42) implies that

$$\bar{c}(\sigma_v^2) \equiv c_1 = \frac{r-n}{RN_1}\left\{F_1 + \mathrm{E}_1[\varphi]\right\}.$$

Substituting the budget constraint (34) and rearranging terms leads to:

$$\bar{c}(\sigma_v^2) = \frac{r-n}{1+n}\left[\frac{F_0+Y_0}{N_0} - c_0(\sigma_v^2) + \frac{1}{RN_0}\mathrm{E}_1[\varphi]\right].$$

It follows that:

$$\bar{\mu} \equiv \mathrm{E}_0[\bar{c}(\sigma_v^2)] = \frac{r-n}{(1+n)N_0}\left[F_0 + Y_0 + \beta\mathrm{E}_0[\varphi] - N_0 c_0(\sigma_v^2)\right], \tag{45}$$

$$\bar{\sigma}^2 \equiv \mathrm{Var}_0[\bar{c}(\sigma_v^2)] = \frac{(r-n)^2}{(1+n)^2N_0^2}\beta^2\mathrm{Var}_0(\mathrm{E}_1[\varphi]). \tag{46}$$

The usual Euler equation for this problem is:

$$u'(c_0(\sigma_v^2)) = \mathrm{E}_0[u'(\bar{c}(\sigma_v^2))],$$

which, after taking a second order Taylor expansion on the right hand side around $\bar{\mu}(\sigma_v^2)$, becomes

$$u'(c_0(\sigma_v^2)) \simeq u'(\bar{\mu}(\sigma_v^2)) + \frac{1}{2}u'''(\bar{\mu}(\sigma_v^2))\bar{\sigma}^2(\sigma_v^2).$$

Implicitly differentiating the latter (approximate) identity with respect to σ_v^2, evaluating at $\sigma_v^2 = 0$ and noting that $\bar{\mu}(0) = c_0(0)$ and $\bar{\sigma}^2(0) = 0$ leads to

$$u''(c_0(0))c_0^1(0) \simeq u''(c_0(0))\bar{\mu}'(0) + \frac{1}{2}u'''(c_0(0))\frac{\partial\bar{\sigma}^2}{\partial\sigma_v^2}(0), \tag{47}$$

where $\bar{\mu}'$ and $\partial\bar{\sigma}^2/\partial\sigma_v^2$ denote the derivatives of $\bar{\mu}$ and $\bar{\sigma}^2$ with respect to σ_v^2. Substituting (45) and (46) in (47) and rearranging terms leads to (44). ∎

Corollary B.2 *Under the same assumptions (and notation) of the preceding proposition, in the case where certainty equivalent consumption does not depend on σ_0^2 and σ_v^2, we have:*

$$\Delta_{BU} = \frac{1}{2}(1+\rho)\frac{\beta(r-n)^2}{(1+n)N_0^2c_0(0,0)^2}\left.\frac{\partial\mathrm{Var}_0\left(Y_0+\mathrm{E}_1[\varphi]\right)}{\partial\sigma_0^2}\right|_{\sigma_v=\sigma_0=0}\sigma_0^2, \tag{48}$$

$$\Delta_{IU} = \frac{1}{2}(1+\rho)\frac{\beta^3(r-n)^2}{(1+n)N_0^2c_0(0,0)^2}\left.\frac{\partial\mathrm{Var}_0\left(Y_0+\mathrm{E}_1[\varphi]\right)}{\partial\sigma_v^2}\right|_{\sigma_v=\sigma_0=0}\sigma_v^2. \tag{49}$$

It follows that

$$\mathrm{Var}[w] = \sum_i c_i^2 \left(e^{2a_i^2\sigma^2} - e^{a_i^2\sigma^2}\right) + 2\sum_{i<j} c_i c_j \left(e^{\frac{1}{2}(a_i+a_j)^2\sigma^2} - e^{\frac{1}{2}(a_i^2+a_j^2)\sigma^2}\right).$$

Differentiating the above expression with respect to σ^2 and evaluating at $\sigma^2 = 0$ leads to (52). ∎

Proposition B.2 *Assume that the logarithm of the price process follows follows a first order autoregressive process:*

$$\log P_t - \mu = \psi(\log P_{t-1} - \mu) + v_t,$$

with the v_t's i.i.d. normal with mean μ_v and variance σ_v^2. We ignore the income effect associated with changes in σ_0 and σ_v. The remainder of the setup is the same as in section 6.1.1.

Then, if $\psi = 1$ the correction factors are given by:

$$\Delta_{BU} = \frac{1}{2}(1+\rho)\frac{R}{(1+n)}\left\{1 + \frac{1-\beta(1+g)}{1-\beta^{T+1}(1+g)^{T+1}}\left[\frac{F_0}{\mu_0}\right]\right\}^{-2} CV_0^2, \tag{53}$$

$$\Delta_{IU} = \frac{1}{2}(1+\rho)\frac{\beta(1+g)^2}{(1+n)}\left\{\frac{1-\{\beta(1+g)\}^T}{[1-\beta(1+g)]\frac{F_0}{\mu_0} + 1 - \{\beta(1+g)\}^{T+1}}\right\}^2 \sigma_v^2, \tag{54}$$

where $CV_0 = \sigma_0/\mu_0$.

If $\psi < 1$ the correction factors are given by:

$$\Delta_{BU} \simeq \frac{1}{2}(1+\rho)\frac{R}{(1+n)}\left\{\frac{\sum_{s=0}^T[\beta\psi(1+g)]^s \exp[(1-\psi^s)(\mu - \log\mu_{P,0})]}{\frac{F_0}{\mu_0} + \sum_{s=0}^T[\beta(1+g)]^s \exp[(1-\psi^s)(\mu - \log\mu_{P,0})]}\right\}^2 CV_0^2, \tag{55}$$

$$\Delta_{IU} \simeq \frac{1}{2}(1+\rho)\frac{R}{(1+n)\psi^2}\left\{\frac{\sum_{t=1}^T[\beta\psi(1+g)]^t \exp[(1-\psi^t)(\mu - \log\mu_{P,0})]}{\frac{F_0}{\mu_0} + \sum_{s=0}^T[\beta(1+g)]^s \exp[(1-\psi^s)(\mu - \log\mu_{P,0})]}\right\}^2 \sigma_v^2. \tag{56}$$

Proof We derive (55), of which (53) is a particular case.[47] The derivation of (54) and (56) is analogous. From (49) it follows that to derive (55)) it suffices to calculate $c_0(0,0)$ and $\partial\mathrm{Var}_0(\mathrm{E}_1[\varphi])/\partial\sigma_v^2$ evaluated at $\sigma_0^2 = \sigma_v^2 = 0$.

From (42) and a slight modification of (51), evaluated at $\sigma_0 = \sigma_v = 0$, we have:

$$c_0(0,0) = \frac{r-n}{RN_0}\left\{F_0 + \mu_0 + \mu_0\sum_{t=0}^{T-1}[\beta(1+g)]^{t+1} e^{(1-\psi^{t+1})(\mu - \log(P_0))}\right\}$$

and hence

$$c_0(0,0) = \frac{r-n}{RN_0}\left\{F_0 + \mu_0\sum_{s=0}^{T}[\beta(1+g)]^s e^{(1-\psi^s)(\mu-\log(P_0))}\right\}. \tag{57}$$

[47] Strictly speaking, L'Hopital's rule must be invoked to go from (55) to (53).

where $f^O = Q_0^O/(Q_0^O + \alpha_1 Q_0^G)$, $f^G = 1 - f^O$ *and* Var_0 *is with respect to the distribution of* Y_0, *assuming* $\sigma_v = 0$, *and*

$$\left.\frac{\partial \mathrm{Var}_0(\mathrm{E}_1[\varphi])}{\partial \sigma_v^2}\right|_{\sigma_v=\sigma_0=0} = \left\{Q_1^O \frac{1-[\beta(1+g^O)]^{T^O}}{1-\beta(1+g^O)} + \alpha_1 Q_1^G \frac{1-[\beta(1+g^G)]^{T^G}}{1-\beta(1+g^G)}\right\}^2 (P_0^O)^2. \quad (61)$$

where Var_0 *is with respect to the distribution of* P_1 *conditional on* P_0, *assuming* $\sigma_0 = 0$.

Expressions (60) and (61) can be used to calculate $c_0^1(0,0)$ *and* $c_0^2(0,0)$ *so as to apply Corollary B.2 to find an approximation for* $c_0(\sigma_0^2, \sigma_v^2)$. ∎

Proof The derivation of (59) is similar to that of (42) because of linearity of the expectations operator. Since the proofs of (61) and (60) are similar, we only provide the latter.

Linearity of the expectations operator and (51) lead to

$$\mathrm{Var}_0(Y_0 + \beta \mathrm{E}_1[\varphi]) = \left\{Q_0^O \frac{1-[\beta(1+g^O)]^{T^O+1}}{1-\beta(1+g^O)} + \alpha_1 Q_0^G \frac{1-[\beta(1+g^G)]^{T^G+1}}{1-\beta(1+g^G)}\right\}^2 \sigma_{P,0}^2. \quad (62)$$

Since

$$\begin{aligned} \sigma_0^2 &= \mathrm{Var}[Y_0] \\ &= \mathrm{Var}[P_0^O Q_0^O + (\alpha_0 + \alpha_1 P_0^O) Q_0^G] \\ &= \mathrm{Var}[P_0^O Q_0^O + \alpha_1 P_0^O Q_0^G] \\ &= [Q_0^O + \alpha_1 Q_0^G]^2 \sigma_{P,0}^2, \end{aligned}$$

the expression obtained in (62) leads to

$$\mathrm{Var}_0(Y_0 + \beta \mathrm{E}_1[\varphi]) = \left\{f^O \frac{1-[\beta(1+g^O)]^{T^O+1}}{1-\beta(1+g^O)} + f^G \frac{1-[\beta(1+g^G)]^{T^G+1}}{1-\beta(1+g^G)}\right\}^2 \sigma_0^2. \quad (63)$$

Differentiating the latter identity with respect to σ_0^2 yields (60). ∎

C Adjustment Costs

Proposition C.1 *Consider the optimal consumption problem with certain income:*

$$\max_{c_t} \sum_{t\geq 0} \tilde{\beta}^t \left[u(c_t) - k(l_t - l_{t-1})^2\right], \quad (64)$$

$$s.t. \sum_{t\geq 0} \beta^t C_t = \mathcal{W}_0, \quad (65)$$

where β *denotes the subjective discount rate, which is assumed equal to the inverse of the gross interest rate* ($R\beta = 1$), *population in period* t *is* $N_t = (1+n)^t$, $\tilde{\beta} = \beta(1+n) < 1$, C_t *denotes period*

Proof This is a well known result, see, for example, Rotemberg (1982) for a considerably more general case. The lower and upper bounds for α in (72) follow from showing that α is increasing in $\tilde{\beta}$ and evaluating α at $\tilde{\beta} = 0$ and $\tilde{\beta} = 1$. ∎

Corollary C.1 *Since there is no income uncertainty, the two preceding propositions can be easily extended to the case of* asymmetric *quadratic adjustment costs, so that:*

$$\text{Cost of adjusting from } l_{-1} \text{ to } l_0 = \begin{cases} k^+(l_t - l_{t-1})^2, & \text{if } l_t > l_{t-1}, \\ k^-(l_t - l_{t-1})^2, & \text{if } l_t < l_{t-1}. \end{cases}$$

Now there will be two values for $\tilde{k}$, $\tilde{k}^+$ and $\tilde{k}^-$, depending on whether per capita consumption increases or decreases. Both of them can be obtained from an expression analogous to (67). The optimal policy continues being of partial adjustment, but the speed of adjustment now depends on whether per capita consumption increases (α^+) or decreases (α^-). Expressions for α^+ and α^- are obtained by substituting $\tilde{k}^+$ and $\tilde{k}^-$ in (70).

Proof Straightforward. ∎

Proposition C.3 *In the setting of the preceding corollary, being indifferent between*

- *the adjustment cost associated this period with an* increase *in per capita expenditure of* $100 \times s_a$ *percent*

and

- *the welfare improvement, in the absence of adjustment costs, associated with a* $100 \times s_{na}$ *percent increase in per capita expenditure*

implies that

$$\tilde{k}^+ \simeq \frac{2s_{na}}{\rho s_a^2}. \tag{73}$$

A similar comparison, with a decrease *in per capita expenditure in the first statement, leads to an analogous expression for $\tilde{k}^-$.*

Proof The welfare loss associated with the first statement is equal to ks_a^2, where we are using the equivalence result in Proposition C.1.

Let $c > c_0$ denote the two per capita consumption levels mentioned in the second statement, and l and l_0 their logarithms. Then

$$\begin{aligned} u(c) - u(c_0) &\simeq u'(c_0)(c - c_0) \\ &= u'(c_0)\left[e^l - e^{l_0}\right] \\ &\simeq u'(c_0)e^{l_0}(l - l_0) \\ &= u'(c_0)c_0 s_{na}. \end{aligned}$$

TABLE 5.1

ADF AND PP TESTS

	1957.I–1999.II	1974.I–1999.II	1986.I–1999.II
ADF no trend	−1.77	−2.60*	−3.42**
ADF with trend	−1.69	−3.83**	−3.52***
PP no trend	−1.65	−2.56	−3.93***
PP with trend	−1.52	−4.57***	−4.25***

Note: *, **, and *** = significant at 10, 5, and 1% respectively.

TABLE 5.2

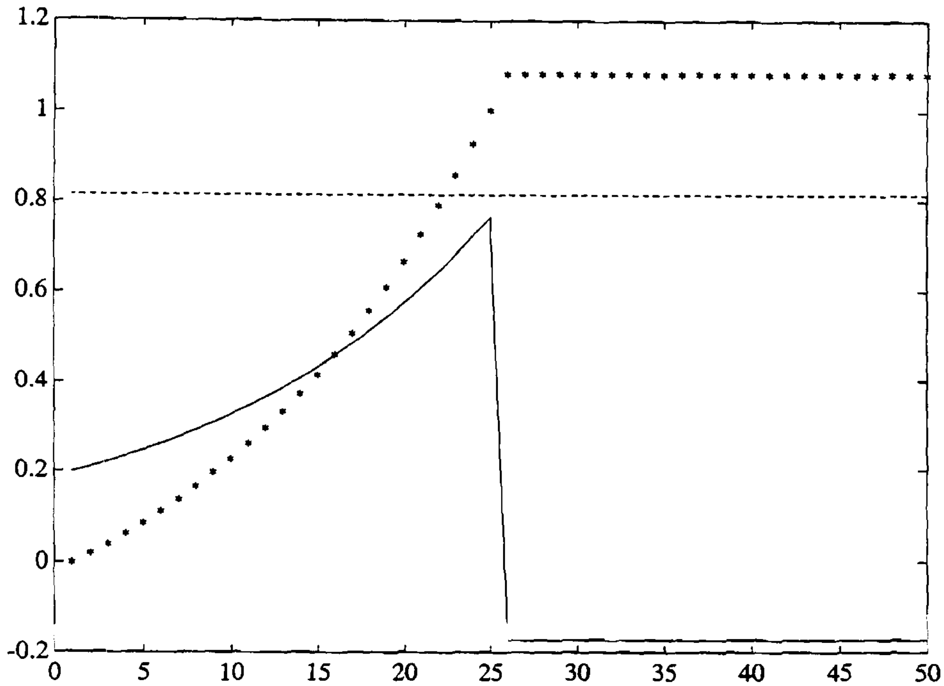
P-VALUES NON-LINEAR ADJUSTMENT TEST

	1957.I–1999.II	1974.I–1999.II	1986.I–1999.II
$d = 1$	0.12 (1)	0.02 (1)	0.10 (1)
$d = 2$	0.56 (1)	0.14 (1)	0.19 (2)
$d = 3$	0.40 (1)	0.40 (1)	0.12 (2)

Note: In parenthesis the value of k that yields white noise.

FIGURE 4.1

Consumption, current account and financial assets with constant non-oil production

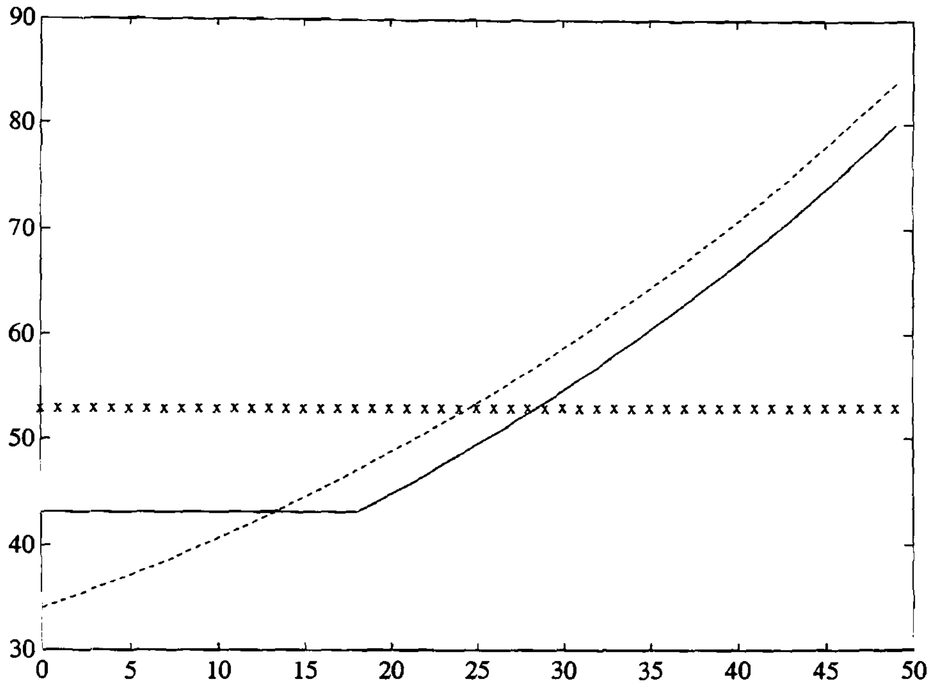


Note to Figure 4.1: The figure shows the optimal paths of normalized consumption (- - -), normalized financial assets (***) and the current account as a fraction of GDP (—) under the assumptions of the benchmark model.

The following assumptions are made: no population growth ($n = 0$), $R = 1.06$, $\beta R = 1$, no initial financial assets ($W_1 = 0$), the optimal mix of the public and private goods requires that the former represent 20% of total consumption, initial oil production, which accrues to the government, accounts for 80% of GDP, while the remaining 20% is produced by the private sector. Oil production remains constant (in real terms) for 25 periods, moment at which oil reserves are exhausted. Production in the non-oil sector remains constant indefinitely.

FIGURE 4.3

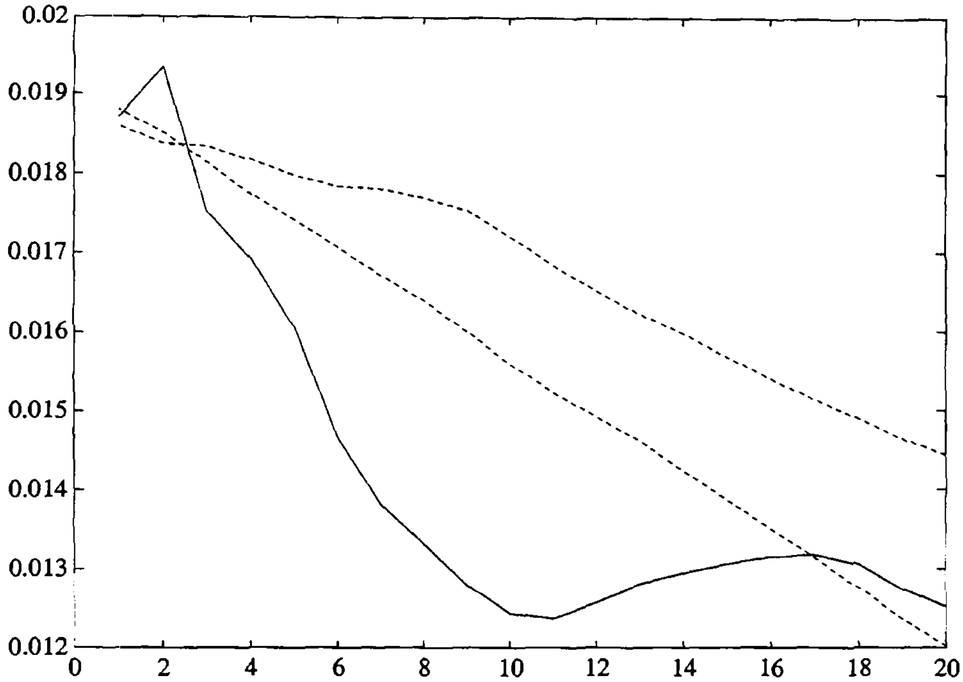
Optimal Consumption Path for Alternative Models with Increasing Non-Oil GDP



Note to Figure 4.3: The figure shows the optimal paths of consumption for the Benchmark Model (xxx), the Permanent Oil Income Model (- - -) and the Conditionally Normative Model (—). Parameter values: no population growth; $R = 1.04$, $\beta R = 1$, initial oil wealth: 100; initial non-oil GDP: 30; non-oil GDP grows 2% per period for 50 periods and then remains constant forever.

FIGURE 5.1

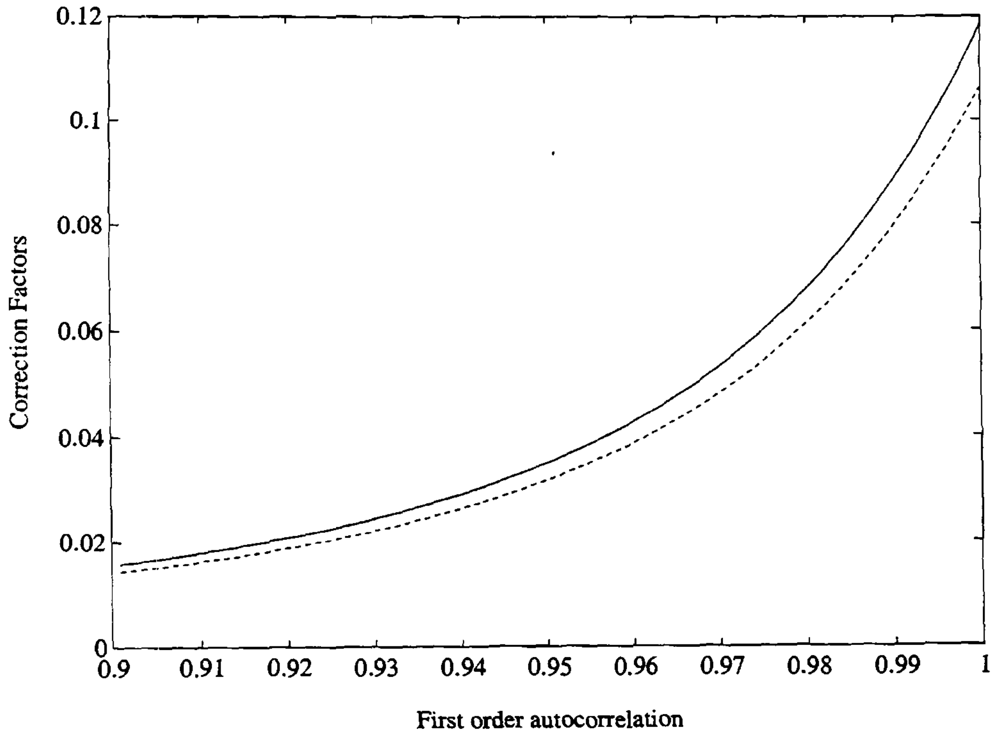
Variance Ratio Test: 1957–1998



Note to Figure 5.1: The figure shows the results of the Variance Ratio tests for the sample 1957–1998 [solid line (—)]. The dashed lines (- - -) show the results of a Montecarlo exercise (with 1000 replications) assuming that the true process is a geometric random walk and a AR(1) with autoregressive coefficient equal to the sample estimate.

FIGURE 6.1

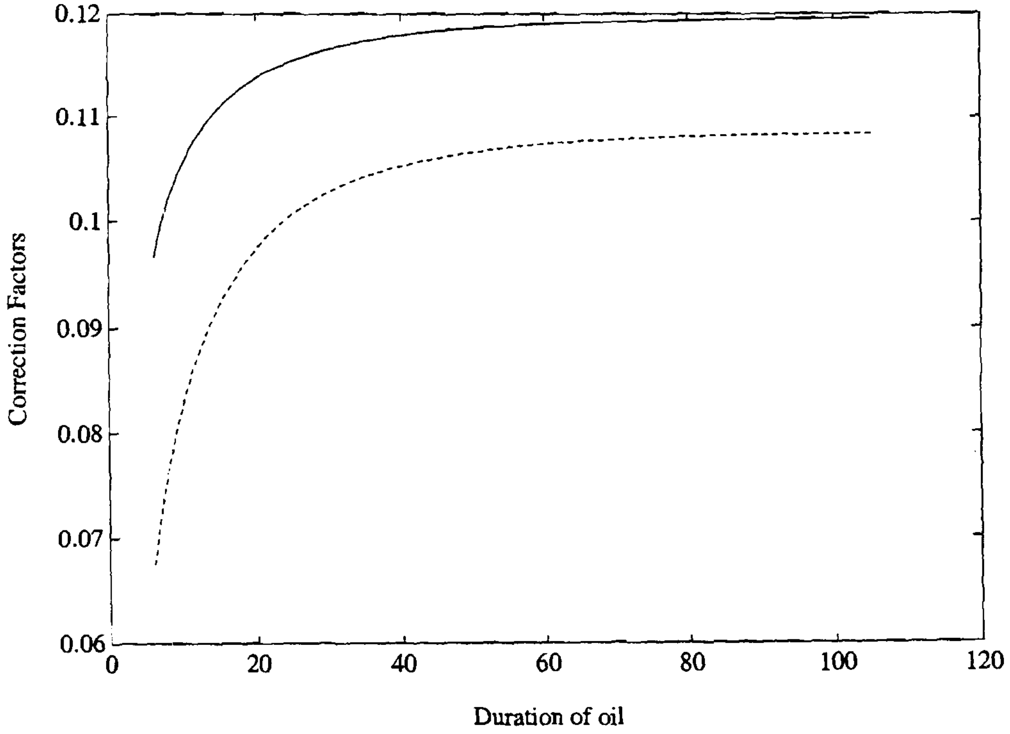
Correction Factors and Shock Persistence



Note to Figure 6.1: The figure shows the correction factors Δ_{BU} (—) and Δ_{IU} (- - -) for different autoregressive coefficients ψ. The rest of the parameters correspond to those of example 6.1.

FIGURE 6.3

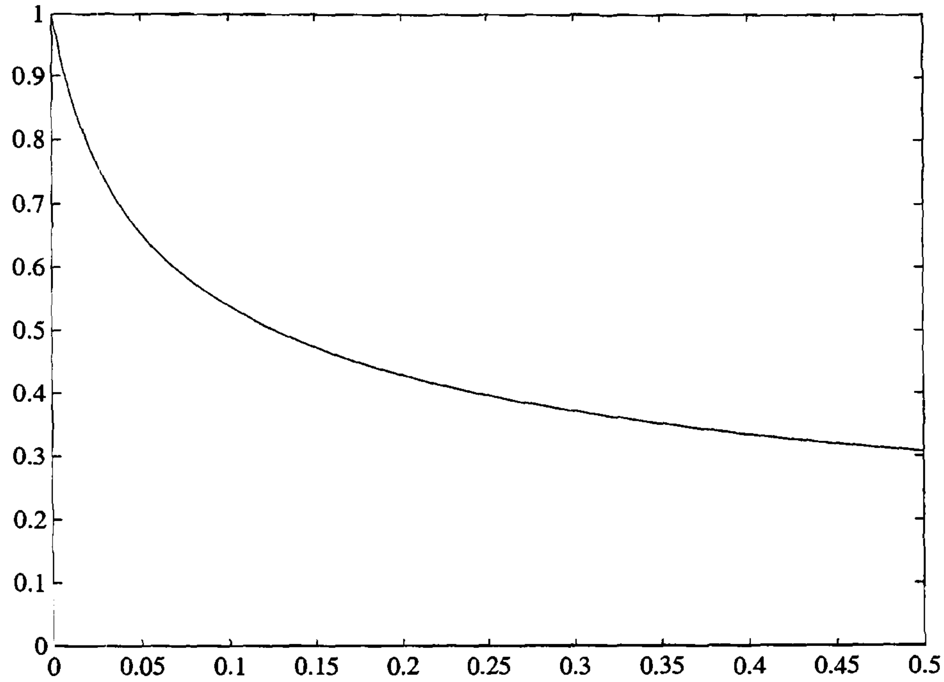
Correction Factors and Resource Duration



Note to Figure 6.3: The figure shows the correction factors Δ_{BU} (—) and Δ_{IU} (- - -) for different resource duration T. The rest of the parameters correspond to those of example 6.1.

FIGURE 6.5

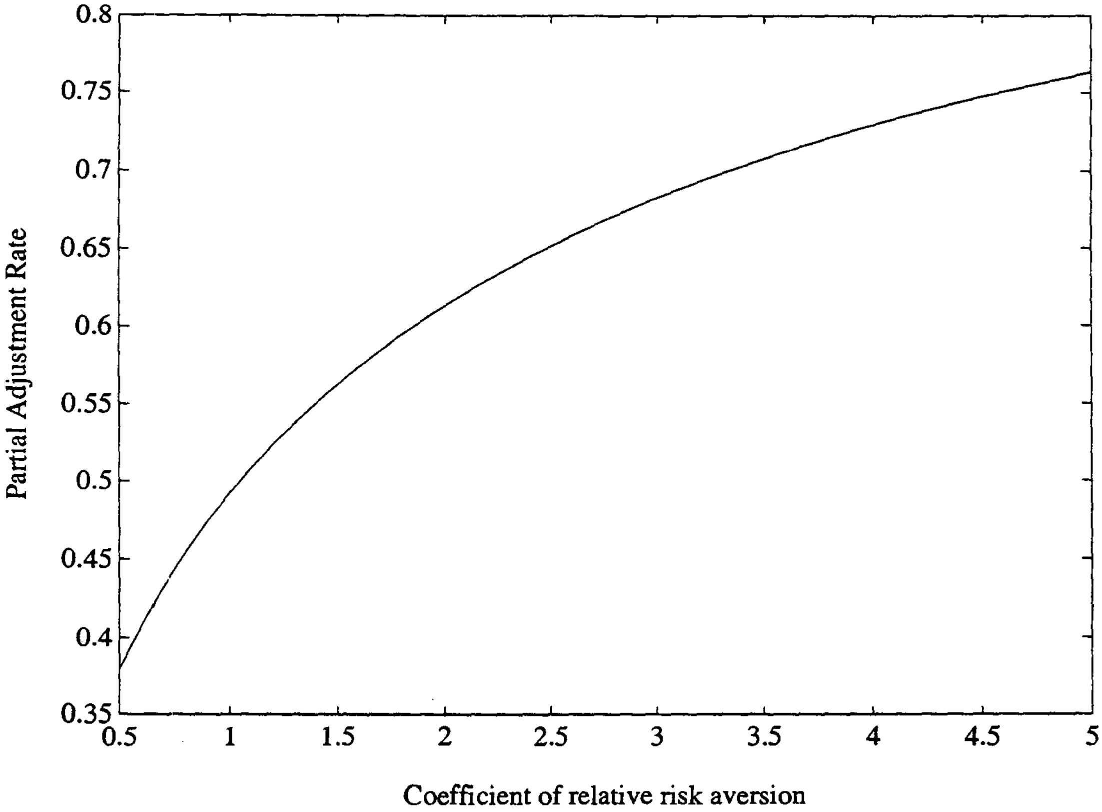
Partial Adjustment Coefficient and Adjustment Cost



Note to Figure 6.5: The figure shows the partial adjustment coefficient for different values of the adjustment cost (s_{na}) for an adjustment (s_a) of 0.20 The rest of the parameters correspond to those of example 6.5.

FIGURE 6.6

Partial Adjustment Coefficient and Risk Aversion



Note to Figure 6.6: The figure shows the partial adjustment coefficient for different values of the coefficient of relative risk aversion (ρ) assuming $s_{na} = 0.04$ and $s_a = 0.20$. The rest of the parameters correspond to those of example 6.5.